
IMO Number Theory Questions

Level: Intermediate **Ref No: M06**

Puzz Points: 13

How many different solutions are there to this word sum, where each letter stands for a different non-zero digit?

$$\begin{array}{r} \text{M A T H S} \\ + \text{M A T H S} \\ \hline \text{C A Y L E Y} \end{array}$$

Level: Intermediate **Ref No: M07**

Puzz Points: 15

- (a) A positive integer N is written using only the digits 2 and 3, with each appearing at least once. If N is divisible by 2 and by 3, what is the smallest possible integer N ?
- (b) A positive integer M is written using only the digits 8 and 9, with each appearing at least once. If M is divisible by 8 and by 9, what is the smallest possible integer M ?

Level: Intermediate **Ref No: M20**

Puzz Points: 10

Show that there are no solutions to this "letter sum". [Each letter stands for one of the digits 0-9; different letters stand for different digits; no number begins with the digit 0.]

$$\begin{array}{r} \text{S E V E N} \\ + \text{O N E} \\ \hline \text{E I G H T} \end{array}$$

Level: Intermediate **Ref No: M22**

Puzz Points: 10

Find the positive integer whose value is increased by 518059 when the digit 5 is placed at each end of the number.

Level: Intermediate **Ref No: M25**

Puzz Points: 15

Find the smallest positive integer which consists only of 0s and 1s when written in base 10, and which is divisible by 12. (*Base 10 just means our standard number system; specifically it means that each place value has ten possible digits, i.e. 0 to 9*)

Level: Intermediate Ref No: M31

Puzz Points: 20

Find the smallest positive multiple of 35 whose digits are all the same as each other.

Level: Intermediate Ref No: M42

Puzz Points: 13

- (a) You are told that one of the integers in a list of distinct positive integers is 97 and that their average value is 47. If the sum of all the integers in the list is 329, what is the largest possible value for a number in the list?
- (b) Suppose the sum of all the numbers in the list can take any value. What would the largest possible number in the list be then?

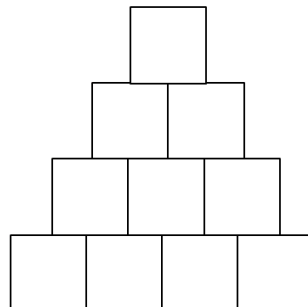
Level: Intermediate Ref No: M43

Puzz Points: 15

Numbers are placed in the blocks shown alongside according to the following two rules.

- (a) For two adjacent blocks in the bottom row, the number in the block to the right is twice the number in the block to the left.
- (b) The number in the block above the bottom row is the sum of the numbers in the two adjacent blocks immediately below it.

What is the smallest positive integer that can be placed in the bottom left-hand block so that the sum of all ten numbers is a cube?



Level: Intermediate Ref No: M44

Puzz Points: 15

Prove that there is exactly one sequence of five consecutive positive integers in which the sum of the squares of the first three integers is equal to the sum of the squares of the other two integers.

Level: Intermediate Ref No: M55

Puzz Points: 10

A palindromic number is one which reads the same when its digits are reversed, for example 23832.

What is the largest six-digit palindromic number which is exactly divisible by 15?

Level: Intermediate Ref No: M59

Puzz Points: 13

Solve the equation $5a - ab = 9b^2$, where a and b are positive integers.

Level: Intermediate Ref No: M63

Puzz Points: 15

A particular four-digit number N is such that:

- (a) The sum of N and 74 is a square; and
- (b) The difference between N and 15 is also a square.

What is the number N ?

Level: Intermediate Ref No: M67

Puzz Points: 20

How many positive integers have a remainder of 31 when divided into 2011?

Level: Intermediate Ref No: M70

Puzz Points: 20

How many solutions are there to the equation $x^2 + y^2 = x^3$, where x and y are positive integers and x is less than 2011?

Level: Intermediate Ref No: M73

Puzz Points: 10

How many four-digit multiples of 9 consist of four different odd digits?

Level: Intermediate Ref No: M76

Puzz Points: 10

The number N is the product of the first 99 positive integers. The number M is the product of the first 99 positive integers after each has been reversed. That is for example, the reverse of 8 is 8; of 17 is 71; and of 20 is 02.

Find the exact value of $N \div M$.

Level: Intermediate Ref No: M84

Puzz Points: 20

All the digits of a certain positive three-digit number are non-zero. When the digits are taken in reverse order a different number is formed. The difference between the two numbers is divisible by eight.

Given that the original number is a square number, find its possible values.

Level: Intermediate Ref No: M86

Puzz Points: 20

Show that the equation:

$$\frac{1}{x} + \frac{1}{y} = \frac{5}{11}$$

Has no solutions for positive integers x, y .

Level: Intermediate Ref No: M96

Puzz Points: 15

Find the possible values of the digits p and q , given that the five-digit number 'p543q' is a multiple of 36.

Level: Intermediate Ref No: M102

Puzz Points: 20

Miko always remembers his four-digit PIN (personal identification number) because

- a) It is a perfect square, and
- b) It has the property that, when it is divided by 2, or 3, or 4, or 5, or 6, or 7, or 8, or 9, there is always a remainder of 1.

What is Miko's PIN?

Level: Intermediate Ref No: M105

Puzz Points: 23

A lottery involves five balls being selected from a drum. Each ball has a different positive integer printed on it.

Show that, whichever five balls are selected, it is always possible to choose three of them so that the sum of the numbers on these three balls is a multiple of 3.

Level: Intermediate Ref No: M107

Puzz Points: 10

The sum of three positive integers is 11 and the sum of the cubes of these numbers is 251.

Find all such triples of numbers.

Level: Intermediate Ref No: M109

Puzz Points: 10

Find all possible solutions to the 'word sum' on the right.

Each letter stands for one of the digits 0-9 and has the same meaning each time it occurs. Different letters stand for different digits. No number starts with a 0.

$$\begin{array}{r} \text{O D D} \\ + \text{O D D} \\ \hline \text{E V E N} \end{array}$$

Level: Intermediate Ref No: M112

Puzz Points: 13

A 'qprime' number is a positive integer which is the product of exactly two different primes, that is, one of the form $q \times p$, where q and p are prime and $q \neq p$.

What is the length of the longest possible sequence of *consecutive* integers all of which are qprime numbers?

Level: Intermediate Ref No: M127

Puzz Points: 10

The five-digit number ' $a679b$ ', where a and b are digits, is divisible by 36. Find all possible such five-digit numbers.

Level: Intermediate Ref No: M133

Puzz Points: 15

An 'unfortunate' number is a positive integer which is equal to 13 times the sum of its digits. Find all 'unfortunate' numbers.

Level: Intermediate Ref No: M136

Puzz Points: 20

From a three-digit number (with no repeated digit and no zero digit) we can form six two-digit numbers by choosing all possible ordered pairs of digits. For example, the number 257 produces the six numbers 25, 52, 57, 75, 27, 72.

Find all such three-digit numbers with no repeated digit for which the sum of the six two-digit numbers is equal to the original three-digit number.

Consider the following three equations:

$$11 - 2 = 3^2$$

$$1111 - 22 = 33^2$$

$$111111 - 222 = 333^2$$

Prove that the pattern suggested by these three equations continues for ever.